5. Work with spaces of continuous functions, equicontinuous families, and apply Weierstrass polynomial approximation to solve related problems.

Contents

Unit I - The Riemann-Stieltjes Integral:
Definition and existence of integrals, Properties of integral, Integration and differentiation, Integration of vector-valued functions, Rectifiable curves. (12 Hours)

Unit II - Sequences and Series of Functions:

Discussion of main problem, Point-wise Convergence and Uniform convergence, Cauchy criterion for Uniform Convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation. (12 Hours)

Unit III – Special Functions and Weierstrass Theorem
The spaces $C(X)$ and $B(X)$, Equicontinuous families of functions, Weierstrass Polynomial approximation theorem and its applications.
Some Special Functions: Power series, The exponential and logarithmic functions, The trigonometric functions, The Gamma function. (12 Hours)

Unit IV - Functions of Several Variables:
Differentiation, Partial Derivatives, Directional derivatives, The Contraction Principle, The Inverse Function Theorem, The Implicit Function Theorem. (12 Hours)

References

[1] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, 1976.

[2] Robert. G. Bartle, *The Elements of Real Analysis*, 2nd Ed., Wiley International Ed., New York, 1976.

[3] Ajith Kumar and S. Kumaresan, *A Basic Course in Real Analysis*, CRC Press, 2014.

[4] Serge Lang, *Analysis* I, Addison Wesley Publishing Company, 1968.

[5] T. M. Apostol, *Mathematical Analysis*, 2nd Ed., Narosa Publishers, 1985.

[6] R. R. Goldberg , *Methods of Real Analysis*, 2nd Ed., Oxford & I. B. H. Publishing Co., New Delhi, 1970.

[7] N. L. Carothers, *Real Analysis*, Cambridge University Press, 2000.

[8] Russel A. Gordon, *Real Analysis - A First Course*, 2nd Ed., Pearson, 2011.

MTH 454	Topology	4 Credits (48 hours)

Course Objectives

1. **Understand Topological Spaces**: Learn the definition of topological spaces, including examples, elementary concepts, and open bases and subbases.
2. **Explore Compactness**: Study compact spaces, product spaces, and Tychonoff's theorem in the context of topology.
3. **Analyze Separation Axioms**: Investigate T1 and Hausdorff spaces, completely regular and normal spaces, and understand Urysohn's lemma, Tietze extension theorem, and Urysohn imbedding theorem.
4. **Examine Connectedness**: Learn about connected spaces, components, totally disconnected spaces, and locally connected spaces.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Identify and work with different types of topological spaces, open bases, and function algebras.
2. Demonstrate understanding of compact spaces and apply Tychonoff's theorem to problems involving product spaces.
3. Distinguish between various separation properties, and apply Urysohn's lemma, Tietze extension theorem, and Urysohn imbedding theorem.
4. Identify and analyze connected, totally disconnected, and locally connected spaces, and understand their components.
5. Use fundamental theorems and concepts to solve problems related to compactness, separation, and connectedness in topological spaces.

Contents

Unit I - Topological Spaces:
The definition and some examples, Open sets, Elementary concepts - Closed sets, Closure of a set, Kuratowski's Closure Axioms, Open bases and open subbases, Lindelof Theorem, Weak topologies, (12 Hours)

Unit II – Function algebras and Compact Spaces:
The function algebras $C(X, R)$ and $C(X, C)$. Compact Spaces, The Heine-Borel Theorem, Product spaces, Tychonoff 's theorem, The Generalized Heine-Borel Theorem, Compactness for metric spaces. (12 Hours)

Unit III - Separation:
T_1-Spaces and Hausdorff spaces, Completely regular spaces and Normal spaces, Urysohn's lemma and Tietze extension theorem, The Urysohn imbedding theorem. (12 Hours)

Unit IV - Connectedness:

Connected spaces, The components of a space, Totally disconnected spaces, Locally connected spaces. (12 Hours)

References

[1] G. F. Simmons, *Introduction to Topology and Modern Analysis*, Tata McGraw-Hill, 2004.

[2] J. R. Munkres, *Topology*, 2nd Ed., Pearson Education, Inc, 2000. (Add first Chapter).