[3] Grimaldi R, *Discrete and Combinatorial Mathematics*, 5th Ed., Pearson, Addison Wesley, 2004.

MTH 452	Algebra - II	4 Credits (48 hours)

Course Objectives

1. **Understand Factoring Techniques:** Study integer and polynomial factorization, including unique factorization domains, Euclidean domains, and irreducibility tests.
2. **Explore Field Extensions:** Learn about field extensions, including the degree of extension, algebraic vs. transcendental elements, and irreducible polynomials.
3. **Examine Advanced Field Concepts:** Understand field isomorphisms, splitting fields, primitive elements, algebraically closed fields, and finite fields.
4. **Apply Galois Theory:** Explore automorphisms, fixed fields, Galois extensions, and the main theorem of Galois theory, including its applications to polynomial equations.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Apply techniques for factoring integers and polynomials, including unique factorization and irreducibility tests.
2. Understand and find the degree of field extensions, and differentiate between algebraic and transcendental elements.
3. Determine isomorphisms between field extensions, and understand concepts like splitting fields and primitive elements.
4. Use Galois theory to analyze automorphisms, fixed fields, and solve cubic, quadratic, and quintic equations.
5. Explore the properties of finite fields and apply field extension concepts to solve related problems.

Contents

Unit I - Factoring:

Factoring Integers, Unique Factorization Domains, Euclidean domains, Content of polynomials, Primitive polynomials, Gauss lemma, Unique factorization in R[x], where R is a UFD, Factoring Integer Polynomials, Irreducibility test mod *p*, Eisenstein's criterion, Gauss primes.

(12 Hours)

Unit II – Fundamentals of Field Extensions

Definition and Examples, Characteristic of a Field, The Degree of Field Extension, Algebraic and Transcendental Elements, Finding the irreducible Polynomial, Ruler and compass constructions.

(12 Hours)

Unit III –Field Extensions and Finite Fields

Isomorphism of field extensions, Adjoining roots, Splitting fields, Primitive elements, Algebraically closed fields, The fundamental theorem of algebra, Finite fields and their properties.

(12 Hours)

Unit IV - Galois Theory:

Automorphisms and Fixed Fields, Galois Extensions, The Main Theorem of Galois Theory, Illustrations of the Main theorem, Cubic Equations, Quadratic Equations, Roots of Unity, Quintic Equations. (12 Hours)

References

[1] Michael Artin, *Algebra,* 2nd Ed., Prentice Hall of India, 2013.

[2] J. B. Fraleigh, *A First Course in Abstract Algebra*, 7th Ed., Addison Wesley, 2003.

[3] I. N. Herstein, *Topics in Algebra*, 2nd Ed., John Wiley & Sons, 2006.

[4] Joseph A. Gallian, *Contemporary Abstract Algebra*, 8th Ed., Cengage Learning India, 2013.

[5] Paul B. Garrett, *Abstract Algebra*, CRC press, 2007.

[6] Thomas W. Hungerford, *Algebra*, Springer, 2004.

[7] David S. Dummit and Richard M. Foote, *Abstract Algebra*, 3rd Ed., Wiley, 2004.

[8] Serge Lang, *Algebra,* 3rd Ed., Springer, 2005.

MTH 453	Real Analysis - II	4 Credits (48 hours)

Course Objectives

1. **Understand Riemann-Stieltjes Integral**: Learn definitions, existence, properties, and applications of the Riemann-Stieltjes integral, including integration of vector-valued functions and rectifiable curves.
2. **Explore Sequences and Series of Functions**: Analyze point-wise and uniform convergence, Cauchy criterion for uniform convergence, and their implications for continuity, integration, and differentiation.
3. **Study Functions of Several Variables**: Master differentiation techniques, partial derivatives, directional derivatives, and key theorems such as the Contraction Principle, Inverse Function Theorem, and Implicit Function Theorem.
4. **Analyze Special functions and Function Spaces**: Understand special functions like exponential, trigonometric functions, gamma functions and their properties, and explore spaces of continuous functions, equicontinuous families, and approximation theorems.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Compute and analyze Riemann-Stieltjes integrals, understanding their properties and applications to vector-valued functions and rectifiable curves.
2. Assess point-wise and uniform convergence of function sequences, and understand their effects on continuity, integration, and differentiation.
3. Use partial derivatives, directional derivatives, and apply the Inverse Function and Implicit Function Theorems to functions of multiple variables.
4. Analyze and work with special functions, including exponential, logarithmic, and trigonometric functions.