Unit II (12 Hours)

Elementary matrices: Elementary matrix operations and elementary matrices, The rank of a matrix and matrix inverses, System of linear equations - theoretical and computational aspects.
Determinants: Determinants of order 2, Determinants of order *n*, Properties of determinants, A characterization of the determinant.

Unit III (12 Hours)

Diagonalization: Eigenvalues and eigenvectors, Diagonalizability, Invariant subspaces and Cyley-Hamilton theorem.
Cyclic subspaces and annihilators, Cyclic decomposition and rational form, The jordan form, Computation of invariant factors.

Unit IV (12 Hours)

Inner product spaces: Recapitulation of inner products and norms, The Gram-Schmidt orthogonalization process and Orthonormal complements.
The adjoint of a linear operator, Normal and self adjoint linear operators, Unitary and orthogonal operators and their matrices, Orthogonal projections and spectral theorem, Bilinear and Quadratic forms, The geometry of orthogonal operators.

References:

1. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence – Linear Algebra, Prentice Hall of India, 4th Edition, 2014.
2. K. Hoffmann and R. Kunz – Linear Algebra, Prentice Hall of India, 2nd Edition, 2013.
3. S. Lang – Linear Algebra, Addison Wesley, London, 1970.
4. Michael Artin – Algebra, Prentice Hall of India, 2rd Edition, 2013.
5. S. Axler - Linear Algebra Done Right, Undergraduate Texts in Mathematics Springer, 4th Edition, 1997.
6. Larry Smith – Linear Algebra, Springer Verlag, 3rd Edition, 1998.
7. Gilbert Strang – Linear Algebra and its Applications, Cengage Learning, 4th Edition, 2006.
8. S. Kumaresan – Linear Algebra - A Geometric Approach, PHI, 2003.

MTH 403	Real Analysis-I	4 Credits (48 hours)

Course Objectives

1. **Understand properties of number systems**: Learn about real and complex number systems, ordered sets, fields, and Euclidean spaces.
2. **Learn basic topology**: Explore metric spaces, open and closed sets, compact sets, and connected sets.
3. **Study numerical sequences and series**: Investigate convergent sequences, series, root and ratio tests, and power series.
4. **Explore continuity and differentiation**: Examine limits, continuous functions, derivatives, mean value theorems, L'Hospital's rule, and Taylor's theorems.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Utilize properties of real and complex numbers, ordered sets, fields, and Euclidean spaces to solve mathematical problems.
2. Define metric spaces, identify open and closed sets, compact sets, perfect sets, and connected sets, and apply the Heine-Borel theorem.

3. Determine convergence of sequences and series using Cauchy sequences, root and ratio tests, and summation by parts.
4. Analyze function limits, identify and categorize discontinuities, and apply concepts of continuity, compactness, and connectedness.
5. Compute derivatives, apply mean value theorems, use L'Hospital's rule, and apply Taylor's theorems to real and vector-valued functions.

Contents

Unit I - The Real and Complex Number System: (12 Hours)

Introduction, Ordered sets, Fields, The real field, The extended real number system, The complex field, Euclidean spaces, Inequalities. Finite, Countable and Uncountable sets, Countability of Rational Numbers.

Unit – II Basic Topology: (12 Hours)

Metric spaces- Definitions, Examples and Basic properties, Open and Closed sets, Compact sets, Hein-Borel Theorem, Perfect sets – Uncountability of real numbers, Connected sets – Discussion in real line under usual metric.

Unit III - Numerical Sequences and Series: (12 Hours)

Convergent sequences, Subsequences, Cauchy sequences, Upper and lower limits, Some special sequences, Series, Series of non-negative terms, The root and ratio tests, Power series, Summation by parts, Absolute convergence.

Unit IV – Continuity and Differentiation: (12 Hours)

Limits of functions, Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities, Monotonic functions, Infinite limits and limits at infinity.

The derivative of a real function, Mean value theorems, The continuity of derivatives, L'Hospital's rule, Derivatives of higher order, Taylor's theorems, Differentiation of vector valued functions.

References

[1] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, 1976.

[2] Robert. G. Bartle, *The Elements of Real Analysis*, 2nd Ed., Wiley International Ed., New York, 1976.

[3] T. M. Apostol, *Mathematical Analysis*, 2nd Ed., Narosa Publishers, 1985.

[4] Ajith Kumar and S. Kumaresan, *A Basic Course in Real Analysis*, CRC Press, 2014.

[5] R. R. Goldberg, *Methods of Real Analysis,* 2nd Ed., Oxford & I. B. H. Publishing Co., New Delhi, 1970.

[6] N. L. Carothers, *Real Analysis*, Cambridge University Press, 2000.

[7] Russel A. Gordon, *Real Analysis - A First Course*, 2nd Ed., Pearson, 2011.