4. Proficiency in determining the ramification index, different of number fields, and splitting of primes, and understanding ideal class groups and their properties.
5. Ability to solve problems related to ideal class groups, Diophantine equations, and apply theoretical results to practical problems in algebraic number theory.

Unit I - Algebraic Numbers (12 Hours)

Recapitulation of Field Extensions and properties, Definition and Examples of algebraic and transcendental numbers, Liouville's Theorem, Algebraic Number Fields, Conjugates and Discriminants, Algebraic Integers, Integral Bases, Norms and Traces, Rings of Integers,Quadratic Fields and Cyclotomic Fields.

Unit II - Factorization into Irreducibles (12 Hours)

Trivial Factorizations, Factorization into Irreducibles, Examples of Non-Unique Factorizationinto Irreducibles, Prime Factorization, Euclidean Domains, Euclidean Quadratic Fields,Consequences of Unique Factorization, The Ramanujan-Nagell Theorem.

Unit III - Factorization of Ideals (12 Hours)

Integral closure, Dedekind domains - Definition, Characterizations, Fractional ideals and unique factorization, Norm of an ideal-Definition and Properties.

Unit IV (12 Hours)

Ramification index and degree of a prime ideal, Different of an algebraic number field, Dedekind's Theorem. Splitting of primes in quadratic fields

The ideal Class Group: Elementary results, Definitions, Finiteness of the ideal class group, Diophantine equations, Supplementary problems

References

[1] Jody Esmonde and M. Ramamurthy, Problems in Algebraic Number Theory, 2nd Ed. Springer Verlag, 2004.

[2] I. N. Stewart and David Tall, Algebraic Number Theory and Fermat's Last Theorem, A. K. Peters Ltd., 2002.

[3] Saban Alaca and Kenneth S. Williams, Introductory Algebraic Number Theory, Cambridge University Press, 2004.

[4] Pierre Samuel, Algebraic Theory of Numbers, Dover Publications, 2008.

[5] Karlheinz Spindler, Abstract Algebra with Applications, Vol. II, Rings and Fields, Marcel Dekkar, Inc, 1994.

MTS 558	Calculus of Variations and Integral Equations	4 Credits (48 hours)

Course Objectives:

1. To introduce the fundamental concepts of variational problems and integral equations.

2. To provide an understanding of solving variational problems with fixed and moving boundaries.
3. To familiarize students with the application of integral equations in solving differential equations.
4. To develop the ability to analyze and solve complex mathematical problems using variational and integral techniques.

Course Outcomes:

1. Understand the foundational principles of the calculus of variations and its applications.
2. Solve variational problems with both fixed and moving boundaries.
3. Apply integral equations to convert and solve differential equations.
4. Analyze and solve Fredholm and Volterra integral equations using various methods.
5. Gain proficiency in the theoretical and practical aspects of integral equations and variational problems.

Contents

Unit I - Variational Problems with the Fixed Boundaries (12 Hours)

Introduction, problem of brachistochrone, problem of geodesics, isoperimetric problem, Variation and its properties, functions and functionals, Comparison between the notion of extrema of a function and a functional. Variational problems with the fixed boundaries, Euler's equation, the fundamental lemma of the calculus of variations, examples, Functionals in the form of integrals, special cases containing only some of the variables, examples, Functionals involving more than one dependent variables and their first derivatives, the system of Euler's equations, Functionals depending on the higher derivatives of the dependent variables, Euler-Poisson equation, examples, Functionals containing several independent variables, Ostrogradsky equation, examples.

Unit II - Variational Problems with Moving Boundaries, Sufficiency Conditions (12 Hours)

Moving boundary problems with more than one dependent variables, transversality condition in a more general case, examples, Extremals with corners, refraction of extremals, examples,One-sided variations, conditions for one sided variations. Field of extremals, central field of extremals, Jacobi's condition, The Weierstrass function, a weak extremum, a strong extremum, The Legendre condition, examples, Transforming the Euler equations to the canonical form, Variational problems involving conditional extremum, examples, constraints involving severalvariables and their derivatives, Isoperimetric problems, examples.

Unit III - Integral Equations (12 Hours)

Introduction, Definitions and basic examples, Classification, Conversion of Volterra Equation to ODE, Conversion of IVP and BVP to Integral Equation. Fredholm's Integral equations - Decomposition, direct computation, Successive approximation, Successive substitution methods for Fredholm Integral Equations.

Unit IV- Voltera Integral Equations and Fredholm's theory (12 Hours)

Voltera Integral Equations: A domain decomposition, series solution, successive approximation, successive substitution method for Volterra Integral Equations, Volterra Integral Equation of first kind, Integral Equations with separable Kernel.

Fredholm's theory: Hilbert-Schmidt Theorem: Fredholm's first, second and third theorem, Integral Equations with symmetric kernel, Eigen function expansion, Hilbert-Schmidt theorem.