Normed linear spaces: Definition and some examples, Banach spaces: Definition and some examples. Some basic properties, Quotient normed spaces. (12 Hours)

Unit II

Continuous linear transformations, Examples, Properties, Extension of Heine-Borel theorem to finite-dimensional normed spaces. The natural embedding of N in N^{**} , The Hahn Banach theorem, The open mapping theorem, Closed graph theorem. (12 Hours)

Unit III

Projections on Banach spaces, Uniform boundedness principle, Conjugate of an operator on a normed linear space.

Hilbert spaces:Definition and examples, Some basic properties, Parallelogram law, Orthogonal complements. (12 Hours)

Unit IV

Orthonormal sets, Complete Orthonormal sets. The conjugate of a Hilbert space, The Adjoint Operator, Self-adjoint operators, Normal and unitary operators, Projections, Finite dimensional spectral theorem, Spectral resolution. (12 Hours)

References

[1] G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 2004.

[2] A. E. Taylor, David Lay, *Introduction to Functional Analysis*, John Wiley and Sons, 1980.

[3] Ward Cheney, *Analysis for Applied Mathematics*, Graduate Texts in Mathematics, Springer, 2001.

[4] Walter Rudin, *Real and Complex Analysis*, 3rd Ed., McGraw Hill, 1986.

[5] M. Thamban Nair, *Functional Analysis* - A First Course, Prentice-Hall, 2002.

MTS 554	Partial Differential Equations	4 Credits (48 hours)

Course Objectives

1. **Explore ODEs in Multiple Variables:** Recap methods for solving differential equations in more than two variables, including Pfaffian differential forms, and study orthogonal trajectories on surfaces.
2. **Understand First Order PDEs:** Learn the origins of first order partial differential equations, methods like Cauchy's characteristics and Charpit's method, and analyze integral surfaces and orthogonal surfaces.
3. **Study Higher Order PDEs:** Analyze linear PDEs with constant coefficients, classify second-order PDEs, and solve elliptic problems using methods for Laplace's equation in various coordinates.
4. **Examine Parabolic and Hyperbolic PDEs:** Investigate diffusion equations and solutions using separation of variables, Dirac Delta function, and study hyperbolic PDEs with methods like D'Alembert's solution and boundary value problems.

Course Outcomes

1. Ability to solve ordinary differential equations in multiple variables using Pfaffian forms and analyze orthogonal trajectories.
2. Proficiency in solving first order PDEs using Cauchy's method, Charpit's method, and understanding integral surfaces and orthogonality.
3. Skill in solving linear PDEs with constant coefficients, classifying second-order PDEs, and applying methods to elliptic differential equations.
4. Competence in solving diffusion equations, using Dirac Delta function, and applying separation of variables in cylindrical and spherical coordinates.
5. Ability to solve hyperbolic PDEs, including one-dimensional wave equations, using canonical reduction, D'Alembert's solution, and boundary value methods.

Unit I

Ordinary differential equations in more than two variables: Recapitulation of Methods of solution of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, Pfaffian differential forms and Pfaffian differential equations and solutions. Orthogonal trajectories of a system of curves on a surface. (12 Hours)

Unit II

First order partial differential equations: Origin of first order partial differential equations, The Cauchy problem for first order equations, Linear equations of first order, Integral surfaces passing through a given curve, Surfaces orthogonal to a given system of surfaces, Nonlinear equations of first order, Cauchy's method of characteristics, Charpit's method, Special types of first order equations. (12 Hours)

Unit III

Higher Order Partial Differential Equations: Linear partial differential equations with constant coefficients, Classification of second order PDE, Canonical forms, Adjoint operators, Riemann's method.

Elliptic Differential Equations: Dirichlet problem for a rectangle, Neumann problem for a rectangle, interior and exterior Dirichlet problem for a circle, Interior Neumann problem for a circle. Solution of Laplace equation in Cylindrical and Spherical coordinates. (12 Hours)

Unit IV

Parabolic Differential Equations: Occurrence of the diffusion equation, Elementary solutions of the diffusion equation, Dirac Delta function, Separation of variables, Solution of diffusion equation in Cylindrical and spherical coordinates.

Hyperbolic Differential Equations: Solution of one dimensional equation by canonical reduction, Initial value problem - D'Alembert's solution, Vibrating string - variable separation method, Forced vibrations, Boundary and initial value problems for two dimensional wave equation, Uniqueness of the solution for the wave equation, Duhamel's principle. (12 Hours)

References