[3] S. Ponnuswamy and H. Silverman, *Complex Variables with Applications*, Birkauser, 2006.

[4] J. B. Conway, *Functions of one Variable*, Narosa, New Delhi, 1996.

[5] B. R. Ash, *Complex Variables*, 2nd Ed., Dover Publications, 2007.

[6] R. V. Churchill, J. W. Brown and R. F. Verlag, *Complex Variables and Applications*, 8th Ed., McGraw Hill, 2009.

MTH 553	Functional Analysis	4 Credits (48 hours)

Course Objectives:

1. **Understand Fundamental Concepts in Metric and Normed Spaces:** Explain the concepts of Baire's theorem, Normed linear spaces and provide examples. Identify and discuss properties of Banach spaces.
2. **Master Theorems and Fundamental Theorems in Functional Analysis:** Understand and utilize the Hahn-Banach theorem, the open mapping theorem, and the closed graph theorem in various contexts.
3. **Explore Advanced Topics in Functional Analysis:** Investigate the properties and applications of Hilbert spaces, including the parallelogram law and orthogonal complements.
4. **Develop Proficiency in Advanced Concepts of Hilbert Spaces and Operators:** Explore the properties of adjoint operators, normal and unitary operators, and projections. Apply the finite-dimensional spectral theorem and spectral resolution techniques.

Course Outcomes:

By the end of the course, students will be able to

1. Demonstrate a solid understanding of Baire's theorem, identifying and classifying normed linear spaces, distinguishing Banach spaces and their essential properties.
2. Apply foundational theorems such as the Heine-Borel theorem adapted to finite-dimensional normed spaces, utilize the Hahn-Banach theorem, open mapping theorem, and closed graph theorem to solve problems in functional analysis.
3. Demonstrate competence in applying the concepts of projections in Banach spaces and analyzing Hilbert spaces, understanding the parallelogram law, and effectively utilizing concepts of orthogonal complements.
4. Students will be able to explain the significance of orthonormal sets and computing adjoint operators, self-adjoint operators, normal and unitary operators, and applying spectral resolution techniques and the finite-dimensional spectral theorem.
5. Finally, students will develop enhanced problem-solving skills in advanced functional analysis, particularly in complex applications involving Hilbert spaces, operators, and spectral theory.

Contents

Unit I

Review of metric spaces: Convergence, Completeness and Baire's theorem.

Normed linear spaces: Definition and some examples, Banach spaces: Definition and some examples. Some basic properties, Quotient normed spaces. (12 Hours)

Unit II

Continuous linear transformations, Examples, Properties, Extension of Heine-Borel theorem to finite-dimensional normed spaces. The natural embedding of N in N^{**} , The Hahn Banach theorem, The open mapping theorem, Closed graph theorem. (12 Hours)

Unit III

Projections on Banach spaces, Uniform boundedness principle, Conjugate of an operator on a normed linear space.

Hilbert spaces:Definition and examples, Some basic properties, Parallelogram law, Orthogonal complements. (12 Hours)

Unit IV

Orthonormal sets, Complete Orthonormal sets. The conjugate of a Hilbert space, The Adjoint Operator, Self-adjoint operators, Normal and unitary operators, Projections, Finite dimensional spectral theorem, Spectral resolution. (12 Hours)

References

[1] G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 2004.

[2] A. E. Taylor, David Lay, *Introduction to Functional Analysis*, John Wiley and Sons, 1980.

[3] Ward Cheney, *Analysis for Applied Mathematics*, Graduate Texts in Mathematics, Springer, 2001.

[4] Walter Rudin, *Real and Complex Analysis*, 3rd Ed., McGraw Hill, 1986.

[5] M. Thamban Nair, *Functional Analysis* - A First Course, Prentice-Hall, 2002.

MTS 554	Partial Differential Equations	4 Credits (48 hours)

Course Objectives

1. **Explore ODEs in Multiple Variables:** Recap methods for solving differential equations in more than two variables, including Pfaffian differential forms, and study orthogonal trajectories on surfaces.
2. **Understand First Order PDEs:** Learn the origins of first order partial differential equations, methods like Cauchy's characteristics and Charpit's method, and analyze integral surfaces and orthogonal surfaces.
3. **Study Higher Order PDEs:** Analyze linear PDEs with constant coefficients, classify second-order PDEs, and solve elliptic problems using methods for Laplace's equation in various coordinates.
4. **Examine Parabolic and Hyperbolic PDEs:** Investigate diffusion equations and solutions using separation of variables, Dirac Delta function, and study hyperbolic PDEs with methods like D'Alembert's solution and boundary value problems.