Curves in R^3, Line Integrals, The Frenet-Serret equations, Geometry of curves in R^3.
(12 Hours)

Unit III

Double integration - Green's theorem, Parametrised surfaces in R^3, Surface area, Surface integrals, Stoke's theorem. (12 Hours)

Unit IV

Triple integrals, The divergence theorem, The geometry of surfaces in R^3, Gaussian Curvature, Geodesic (8 Hours)

References

[1] Sean Dineen, *Multivariate Calculus and Geometry*, 3rd Ed., Springer Undergraduate Mathematics Series, 2014.

[2] Andrew Pressly, *Elementary Differential Geometry*, 2nd Ed., Springer Undergraduate Mathematics Series, 2010.

[3] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, New York, 1976.

[4] J. A. Thorpe, *Elementary Topics in Differential Geometry*, Undergraduate Texts in Mathematics, Springer Verlag, 1994.

[5] W. Klingenberg, *A course in Differential Geometry*, Springer Verlag, 1983.

MTS 505	Advanced Numerical Analysis	4 Credits (48 hours)

Course Objectives

1. **Master Numerical Integration**: Gain a comprehensive understanding of various numerical integration techniques, including those based on interpolation and undetermined coefficients, and specialized methods such as Romberg integration and Gauss quadrature methods.
2. **Solve Ordinary Differential Equations Numerically**: Develop proficiency in numerical methods for solving ordinary differential equations (ODEs), including Euler, Taylor series, Runge-Kutta, and multistep methods, as well as methods for boundary value problems.
3. **Analyze Systems of Linear Differential Equations**: Understand and apply numerical techniques for solving systems of linear differential equations, including finite difference methods and explicit schemes for different types of partial differential equations.
4. **Implement and Analyze Algorithms**: Enhance skills in implementing numerical algorithms and analyzing their accuracy, stability, and efficiency, particularly in the context of solving differential equations and integration problems.

Course Outcomes

1. Ability to implement and apply various numerical integration methods, including Romberg, Gauss-Legendre, Gauss-Chebyshev, Gauss-Laguerre, and Gauss-Hermite methods, as well as double integration using the trapezoidal and Simpson's rules.
2. Proficiency in solving ordinary differential equations using numerical methods such as Euler, Backward Euler, Mid-point, Taylor series, and Runge-Kutta methods, and addressing boundary value problems using finite difference and predictor-corrector methods.

3. Competence in solving systems of linear differential equations using difference methods for parabolic, hyperbolic, and elliptic equations, and understanding the application of schemes like Schmidt, Du Fort-Frankel, Crank-Nicolson, and Crandall.
4. Skill in implementing and analyzing numerical algorithms for differential equations and integration, with a focus on accuracy, stability, and computational efficiency.
5. Enhanced problem-solving abilities in applying numerical methods to practical and theoretical problems involving integration and differential equations, and the ability to critically evaluate and choose appropriate methods for specific problems.

Contents

Unit I - Numerical Integration:

Recapitulation of the methods based on interpolation, Methods based on undetermined coefficients. Romberg integration, Gauss-Legendre integration methods, Gauss-Chebyshev integration methods, Gauss-Laguerre integration methods, Gauss-Hermite integration methods. Double integration, Trapezoidal rule, Simpson's rule. (15 Hours)

Unit II - Ordinary Differential Equations:

Introduction, Numerical methods, Euler method, Backward Euler method, Mid-point method, Single step methods, Taylor series method, Runge-Kutta methods, Multistep methods, Determination of a_j and b_j, Predictor-corrector methods, Boundary value problems, Finite difference methods, Trapezoidal, Dahlquist and Numerov methods. (15 Hours)

Unit III - Systems of Linear Differential Equations:

Introduction, Difference methods, Parabolic equations in one space dimension, Schmidt formula, Du Fort-Frankel scheme, Crank-Nicolson and Crandall schemes, Solution of hyperbolic equation in one dimension by explicit schemes, The CFL condition, Elliptic equations, Dirichlet problem, Neumann problem, Mixed problem. (18 Hours)

References

[1] M. K. Jain, S. R. K. Iyengar, P. K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 6th Ed., New Age International, 2012.

[2] C. F. Gerald and P. O. Wheatly, *Applied Numerical Analysis*, Pearson Education, Inc., 1999.

[3] M. K. Jain, *Numerical Solution of Differential Equations*, 2nd Ed., New Age International (P) Ltd., New Delhi, 1984.

[4] A. R. Mitchell, *Computational Methods in Partial Differential Equations*, John Wiley and Sons, Inc., 1969.

MTS 506	Commutative Algebra	4 Credits (48 hours)

Course Objectives

1. **Understand Rings and Ideals:** Develop a solid understanding of the fundamental concepts in ring theory, including zero divisors, nilpotent elements, units, and various types of ideals (prime, maximal, nilradical, Jacobson radical).