Differentiation Integration: Continuity of monotone functions, Differentiability of monotone functions, Functions of bounded variation, Absolute continuity, Integrating derivatives: Differentiating indefinite integrals.

The L^p Spaces: Normed linear spaces, the inequalities of Young, Holder and Monkowski, Completeness of L^p Spaces. (12 Hours)

References

[1] H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, 4th Ed., Pearson, 2015.

[2] H. L. Royden, *Real Analysis*, 3rd Ed., Prentice - Hall, 2003.

[3] G. D. Barra, *Introduction to Measure Theory*, Van Nostrand Reinhold Company Ltd., 1974.

[4] Walter Rudin, *Real and Complex Analysis*, 3rd Ed., Tata McGraw Hill Publishing Company, 1987.

[5] P. R. Halmos, *Measure Theory*, Springer Verlag, 1974.

[6] F. Hewitt and K. Stromberg, *Real and Abstract Analysis*, Springer Verlag, 1965.

[7] Inder K. Rana, *An Introduction to Measure and Integration*, 2nd Ed., Narosa Publishing House, 1997.

MTH 504	Multivariate Calculus and Geometry	4 Credits (48 hours)

Course Objectives

1. **Understanding Differentiable Functions**: Develop a deep understanding of differentiable functions, including the concepts of level sets, tangent spaces, and optimization techniques like Lagrange multipliers.
2. **Exploring Curves and Their Geometry**: Gain insights into the geometry of curves in R^3, including the study of line integrals and the Frenet-Serret equations.
3. **Mastering Multiple Integrals**: Achieve proficiency in double and triple integrals, including their applications in Green's theorem, Stoke's theorem, and the divergence theorem.
4. **Studying Surfaces and Their Properties**: Understand the geometry of surfaces in R^3, including parametrized surfaces, surface area, surface integrals, Gaussian curvature, and geodesics.

Course Outcomes

Upon successful completion of this course, students will gain the following:

1. Ability to apply optimization techniques, including Lagrange multipliers, to find maxima and minima on open sets.
2. Proficiency in understanding and analyzing curves in R^3 using line integrals and the Frenet-Serret equations, and interpreting the geometric properties of these curves.
3. Ability to compute double and surface integrals and apply Green's theorem and Stoke's theorem to solve problems involving parametrized surfaces in R^3.
4. Competence in evaluating triple integrals and applying the divergence theorem, and understanding their geometric interpretations.
5. Understanding the geometry of surfaces in R^3, including concepts like Gaussian curvature and geodesics, and their applications in various mathematical and physical contexts.

Unit I

Introduction to differentiable functions-, Level sets and tangent spaces, Lagrange multipliers, Maxima and minima on open sets. (12 Hours)

Unit II

Curves in R^3, Line Integrals, The Frenet-Serret equations, Geometry of curves in R^3.
(12 Hours)

Unit III

Double integration - Green's theorem, Parametrised surfaces in R^3, Surface area, Surface integrals, Stoke's theorem. (12 Hours)

Unit IV

Triple integrals, The divergence theorem, The geometry of surfaces in R^3, Gaussian Curvature, Geodesic (8 Hours)

References

[1] Sean Dineen, *Multivariate Calculus and Geometry*, 3rd Ed., Springer Undergraduate Mathematics Series, 2014.

[2] Andrew Pressly, *Elementary Differential Geometry*, 2nd Ed., Springer Undergraduate Mathematics Series, 2010.

[3] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, New York, 1976.

[4] J. A. Thorpe, *Elementary Topics in Differential Geometry*, Undergraduate Texts in Mathematics, Springer Verlag, 1994.

[5] W. Klingenberg, *A course in Differential Geometry*, Springer Verlag, 1983.

MTS 505	Advanced Numerical Analysis	4 Credits (48 hours)

Course Objectives

1. **Master Numerical Integration**: Gain a comprehensive understanding of various numerical integration techniques, including those based on interpolation and undetermined coefficients, and specialized methods such as Romberg integration and Gauss quadrature methods.
2. **Solve Ordinary Differential Equations Numerically**: Develop proficiency in numerical methods for solving ordinary differential equations (ODEs), including Euler, Taylor series, Runge-Kutta, and multistep methods, as well as methods for boundary value problems.
3. **Analyze Systems of Linear Differential Equations**: Understand and apply numerical techniques for solving systems of linear differential equations, including finite difference methods and explicit schemes for different types of partial differential equations.
4. **Implement and Analyze Algorithms**: Enhance skills in implementing numerical algorithms and analyzing their accuracy, stability, and efficiency, particularly in the context of solving differential equations and integration problems.

Course Outcomes

1. Ability to implement and apply various numerical integration methods, including Romberg, Gauss-Legendre, Gauss-Chebyshev, Gauss-Laguerre, and Gauss-Hermite methods, as well as double integration using the trapezoidal and Simpson's rules.
2. Proficiency in solving ordinary differential equations using numerical methods such as Euler, Backward Euler, Mid-point, Taylor series, and Runge-Kutta methods, and addressing boundary value problems using finite difference and predictor-corrector methods.