2. Apply topological concepts of the complex plane to understand the continuity and differentiability of complex functions.
3. Solve problems involving differentiability, Cauchy-Riemann equations, and harmonic functions.
4. Evaluate complex line integrals using the Cauchy-Goursat theorem and understand the concept of winding number.
5. Compute integrals using Cauchy's integral formula, understand Morera's theorem, and develop Taylor and Laurent series.

Contents

Unit I

Recapitulation of Complex numbers, Geometric interpretation, Square roots.
Rational powers of a complex number, Topology of the complex plane, One-to-one and onto functions, Concepts of limit and continuity, Stereographic projection, Sequences and series of functions.

(12 Hours)

Unit II

Differentiability and Cauchy-Riemann equations, Harmonic functions, Power series as an analytic function, Exponential and trigonometric functions, Logarithmic function, Inverse functions.

(12 Hours)

Unit III

Curves in the complex plane, Properties of complex line integrals, Cauchy-Goursat theorem, Consequence of simply connectivity, Winding number or index of a curve, Homotopy version of Cauchy's theorem. (12 Hours)

Unit IV

Cauchy integral formula and computation of line integrals, Morera's theorem, Existence of harmonic conjugate, Taylor's theorem, Zeros of analytic functions, Laurent series.

(12 Hours)

References:

1. S. Ponnusamy - Foundations of Complex Analysis, Narosa, New Delhi, 2nd edition, 2005.
2. S. Ponnuswamy and H. Silverman – Complex Variables with Applications, Birkhäuser, 2006.
3. J. B. Conway – Functions of one Variable, Narosa, New Delhi, 1996.
4. Lars V. Ahlfors – Complex Analysis, McGraw Hill, 3rd Edition, 1979.
5. 2. B. R. Ash – Complex Variables, Dover Publications, 2nd Edition, 2007.
6. R. V. Churchill, J. W. Brown and R. F. Verlag – Complex Variables and Applications, Mc Graw Hill, 8th Edition, 2009.

MTH 503	Measure and Integration	4 Credits (48 hours)

Course Objectives

1. **Understand Fundamental Concepts**: Gain a comprehensive understanding of the fundamental concepts in Lebesgue measure theory, including outer measures, σ-algebras, and measurable sets.

2. **Develop Analytical Skills**: Develop the ability to perform rigorous analysis using concepts such as countable additivity, continuity, and key theorems like the Borel-Cantelli lemma and Egoroff's theorem.
3. **Master Integration Techniques**: Achieve proficiency in the Riemann and Lebesgue integration techniques, understanding their applications and distinctions, and mastering the integration of various classes of functions.
4. **Explore Advanced Topics**: Explore advanced topics such as differentiation and integration of monotone functions, functions of bounded variation, and the properties and completeness of L^p spaces.

Course Outcomes

Upon successful completion of this course, students will gain the following :

1. Ability to define and work with Lebesgue measurable sets and functions, including performing operations such as sums, products, compositions, and approximations.
2. Proficiency in computing the Lebesgue integral for different types of functions, including bounded measurable functions and non-negative functions over sets of finite measure.
3. Understanding and application of advanced theorems related to integration, such as the Vitali convergence theorem, and the ability to distinguish between Riemann and Lebesgue integrability.
4. Ability to analyze the continuity and differentiability of monotone functions, work with functions of bounded variation, and understand the process of integrating derivatives and differentiating indefinite integrals.
5. Comprehension of the structure and properties of L^p spaces, including norms, inequalities, and the completeness of these spaces, and the ability to apply these concepts to solve relevant mathematical problems.

Unit I

Introduction to Lebesgue measure, Lebesgue outer measure, The σ-algebra of Lebesgue measurable sets, Outer and inner approximation of Lebesgue measurable sets, Countable additivity, continuity and the Borel-Cantelli lemma, (12 Hours)

Unit II

Non-measurable stes, Lebesgue measurable functions- Sums, products and compositions, Sequential point-wise limits, Simple approximation, Littilewood's three principles, Egoroff's theorem, Lusin's theorem, Riemann integral, Lebesgue integral of a bounded measurable function over a set of finite measure. (12 Hours)

Unit III

Lebesgue integral of a measurable non-negative function, General Lebesgue integral, Countable additivity and continuity of integration, Uniform integrability: The Vitali convergence theorem, Charaterizations of Riemann and Lebesgue integrability. (12 Hours)

Unit IV

Differentiation Integration: Continuity of monotone functions, Differentiability of monotone functions, Functions of bounded variation, Absolute continuity, Integrating derivatives: Differentiating indefinite integrals.

The L^p Spaces: Normed linear spaces, the inequalities of Young, Holder and Monkowski, Completeness of L^p Spaces. (12 Hours)

References

[1] H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, 4th Ed., Pearson, 2015.

[2] H. L. Royden, *Real Analysis*, 3rd Ed., Prentice - Hall, 2003.

[3] G. D. Barra, *Introduction to Measure Theory*, Van Nostrand Reinhold Company Ltd., 1974.

[4] Walter Rudin, *Real and Complex Analysis*, 3rd Ed., Tata McGraw Hill Publishing Company, 1987.

[5] P. R. Halmos, *Measure Theory*, Springer Verlag, 1974.

[6] F. Hewitt and K. Stromberg, *Real and Abstract Analysis*, Springer Verlag, 1965.

[7] Inder K. Rana, *An Introduction to Measure and Integration*, 2nd Ed., Narosa Publishing House, 1997.

MTH 504	Multivariate Calculus and Geometry	4 Credits (48 hours)

Course Objectives

1. **Understanding Differentiable Functions**: Develop a deep understanding of differentiable functions, including the concepts of level sets, tangent spaces, and optimization techniques like Lagrange multipliers.
2. **Exploring Curves and Their Geometry**: Gain insights into the geometry of curves in R^3, including the study of line integrals and the Frenet-Serret equations.
3. **Mastering Multiple Integrals**: Achieve proficiency in double and triple integrals, including their applications in Green's theorem, Stoke's theorem, and the divergence theorem.
4. **Studying Surfaces and Their Properties**: Understand the geometry of surfaces in R^3, including parametrized surfaces, surface area, surface integrals, Gaussian curvature, and geodesics.

Course Outcomes

Upon successful completion of this course, students will gain the following:

1. Ability to apply optimization techniques, including Lagrange multipliers, to find maxima and minima on open sets.
2. Proficiency in understanding and analyzing curves in R^3 using line integrals and the Frenet-Serret equations, and interpreting the geometric properties of these curves.
3. Ability to compute double and surface integrals and apply Green's theorem and Stoke's theorem to solve problems involving parametrized surfaces in R^3.
4. Competence in evaluating triple integrals and applying the divergence theorem, and understanding their geometric interpretations.
5. Understanding the geometry of surfaces in R^3, including concepts like Gaussian curvature and geodesics, and their applications in various mathematical and physical contexts.

Unit I

Introduction to differentiable functions-, Level sets and tangent spaces, Lagrange multipliers, Maxima and minima on open sets. (12 Hours)

Unit II